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ROLE OF DYNAMIC MODELING IN THE RESEARCH OF THE SHADOW ECONOMY

Dynamic modeling of the shadow economy is becoming an integral part of modern economic analytics. It allows you to take into account the complex interrelationships and dynamics of the phenomenon of the shadow economy, which is critical for analyzing its impact on global financial and economic processes in the country and beyond. Using real-time data and intelligent algorithms helps identify changes and trends, which is key to predicting and managing in a changing economic environment. The purpose of this study is to consider and structure four types of dynamic mathematical models aimed at analyzing and forecasting the phenomena of the shadow economy of Ukraine in order to improve policies and management strategies aimed at reducing the negative impact of the phenomenon of informal economic activity on the economy of Ukraine and society. As a result of the research, the main methods of such dynamic models were considered: agent models (ABM), dynamic stochastic models of general equilibrium (DSGE), macroeconomic models, time series models; and how they relate to the shadow economy, and a comparative analysis of the application of these types of models to shadow economy simulations was conducted.

Keywords: *shadow economy; mathematical modeling; dynamic models; econometric methods; time series.*

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РОЛЬ ДИНАМІЧНОГО МОДЕЛЮВАННЯ У ДОСЛІДЖЕННІ ТІНЬОВОЇ ЕКОНОМІКИ

Динамічне моделювання тіньової економіки стає невід'ємною складовою сучасної економічної аналітики. Воно дозволяє враховувати складні взаємозв'язки та динаміку явища тіньової економіки, що є критичним для аналізу її впливу на глобальні фінансові та економічні процеси в країні та поза її межами. Використання даних в режимі реального часу та інтелектуальних алгоритмів допомагає виявляти зміни та тенденції, що є ключовим для прогнозування та управління у змінному економічному середовищі. Метою цього дослідження є розгляд та структурування чотирьох типів динамічних математичних моделей, спрямованих на аналіз та прогнозування явищ тіньової економіки України з метою вдосконалення політики та стратегій управління, які мають на меті зменшення негативного впливу явища неформальної економічної діяльності на економіку України та суспільство. В результаті проведеного дослідження було розглянуто головні методи таких динамічних моделей: агентні моделі (ABM), динамічні стохастичні моделі загальної рівноваги (DSGE), макроекономічні моделі, моделі часових рядів; а також, як вони пов'язані з тіньовою економікою, а також був проведений порівняльний аналіз застосування цих типів моделей для моделювання тіньової економіки.

Ключові слова: *тіньова економіка; математичне моделювання; динамічні моделі; економетричні методи; часові ряди.*

Formulation of the problem. Dynamic modeling plays a crucial role in understanding the shadow economy within the broader context of modern economics. By utilizing advanced technologies like mathematical modeling, researchers can delve into complex economic processes and risks with precision and efficiency. These dynamic models consider numerous factors and their interplay, enhancing our comprehension of economic phenomena, including the shadow economy's intricacies. They facilitate the identification of patterns and correlations through vast data analysis and artificial intelligence applications, aiding in accurate forecasting and strategic decision-making. The integration of modern management technologies further boosts competitiveness and efficiency in the global economic landscape. Thus, the emphasis on dynamic modeling in studying the shadow economy reflects its pivotal role in addressing the unique challenges posed by undeclared transactions and tax evasion. Such structured models offer nuanced insights and predictive capabilities, essential for informed policy-making and effective management strategies. Overall, dynamic modeling is instrumental in refining our understanding and response to the complexities of the shadow economy, aligning with the increasing demand for precise and practical analytical tools in economic research and policymaking. Structuring dynamic modeling is an important step for its continued successful use for several reasons. First, a clear structure allows you to systematize and organize complex data and processes, which makes the model more understandable and accessible for analysis. Secondly, structured models allow more accurate consideration of relationships between various variables and factors, which increases their adequacy in forecasting and problem solving. In addition, structured dynamic modeling facilitates further refinement and modification of the model over time, as it allows for a faster response to changes in the economic environment and data. This approach also helps preserve information and knowledge for future research and analysis, which is important for the scientific and practical community.

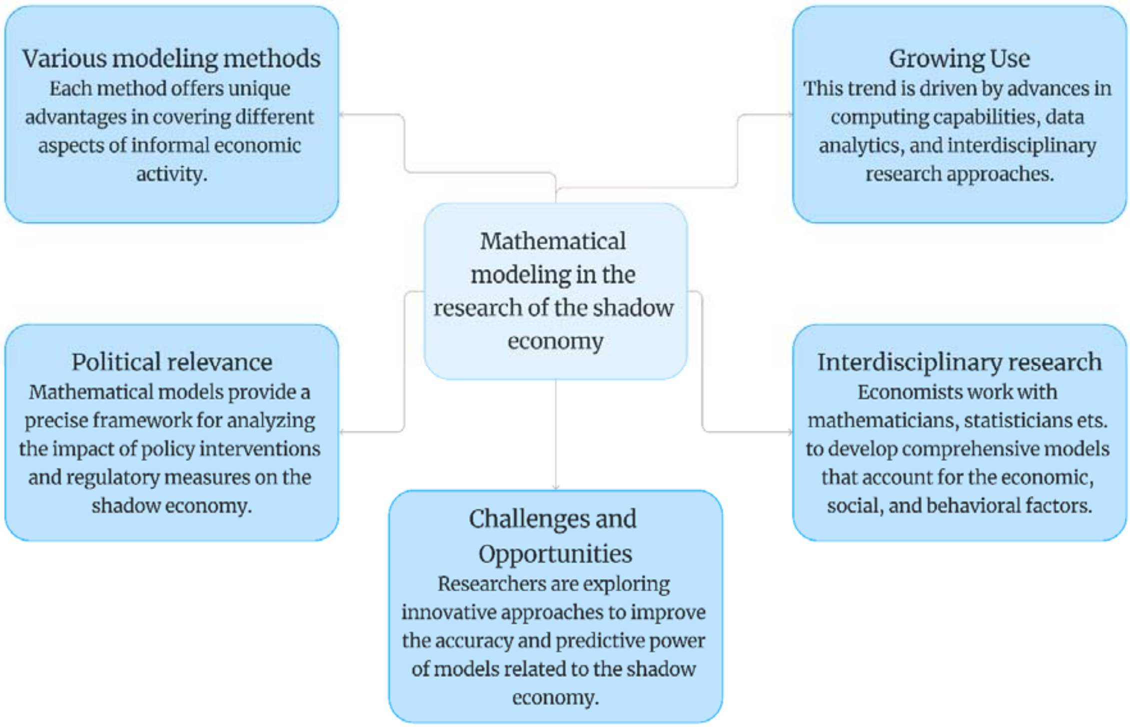
Analysis of recent research and publications. Many scientists are engaged in the dynamic modeling of the shadow economy, because this is an urgent issue. Patrick Fève, Alban Moura, Olivier Pierrard studied the issue of shadow banking and financial regulation using the DSGE model [1]. Celso J. Costa Junior, Alejandro C. Garcia-Cintado and Carlos Usabiaga also used the DSGE model to study fiscal adjustments and the shadow economy in an emerging market [2]. Sascha Hokamp and Götz Seibold used agent-based modeling (ABM) to study how much rationality tolerates the shadow economy [3]. Macroeconometric modeling was done by Roberto Dell'Anno, Miguel Gómez-Antonio and Angel Pardo, who used the MIMIC model to study The shadow economy in three Mediterranean countries: France, Spain and Greece [4]. Piotr Dybka, Michał Kowalczuk, Bartosz Olesiński, Andrzej Torój and Marek Rozkrut [5] also used the econometric MIMIC model for their research, as well as Ligita Gasparėnienė, Rita Remeikienė, Romualdas Ginevičius and Martin Schieg [6]. Iustina Alina Boitan and Sorina Emanuela Ștefoni used the ARIMA time series model to study impact assessment and policy implications of the digitalization and the shadow economy [7].

The aim of the study. The purpose of this study is to consider and structure four types of dynamic mathematical models aimed at analyzing and forecasting the phenomena of the shadow economy of Ukraine in order to improve policies and management strategies aimed at reducing the negative impact of the phenomenon of informal economic activity on the economy of Ukraine and society.

Research methods. A literature review method was used to gather information on the types of dynamic models in order to understand the theory and methodology related to dynamic modeling in the context of the shadow economy. A comparative analysis method was used for four types of dynamic models to evaluate their strengths and weaknesses, applicability, and effectiveness. The synthesis method was used to identify the most important features, as the study of dynamic models is important for the analysis of the shadow economy of Ukraine.

Main material.

Why is mathematical modeling important for studying the shadow economy of Ukraine? Mathematical modeling is increasingly used to study the shadow economy due to its structured and quantitative insights into complex economic phenomena. Over recent decades, its usage in economic research has surged, driven by advancements in computing and interdisciplinary methods. Researchers employ various modeling techniques like econometrics, game theory, and agent-based modeling to explore different facets of the shadow economy. Despite their benefits, these methods have limitations, as discussed later. Political interest in understanding and predicting the shadow economy has also fueled mathematical modeling's adoption. Collaboration across disciplines, including economics, mathematics, statistics, and computer science, ensures comprehensive models that consider economic, social, and behavioral factors [8]. Challenges such as data constraints and model complexity persist, prompting researchers to explore innovative solutions like big data analytics and artificial intelligence integration to enhance model accuracy and prediction capabilities in shadow economy studies. The conceptual map in Fig. 1 shows some key points regarding the peculiarities of using mathematical modeling in the study of the shadow economy at the present moment.

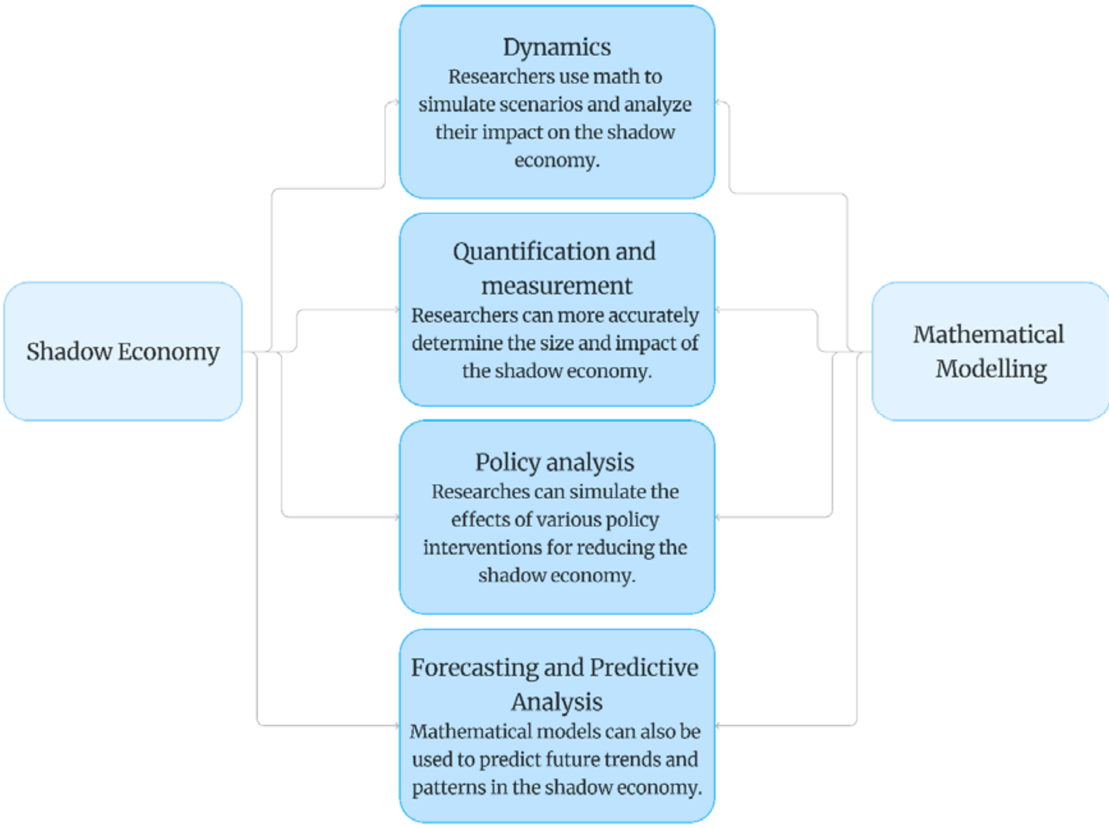


Source: author's own research.

Fig. 1. **Concept map on the peculiarities of using mathematical modeling in the research of the shadow economy**

Mathematical modeling provides crucial insights into the dynamics of the shadow economy, including its size, growth rate, and influencing factors. By formulating equations based on variables like informal economic activities and tax evasion, researchers can simulate scenarios and analyze their impact. This enables a more accurate understanding of the shadow economy's scale and effects, aiding decision-making and policy formulation. Models also help assess policy interventions' effectiveness, like tax reforms or regulatory measures, and predict future trends, guiding strategic planning. Fig. 2 shows the connection between the shadow economy and mathematical modeling with the help of a concept map.

From a political point of view, mathematical modeling of the shadow economy is important for Ukraine for many reasons. After all, mathematical models provide an assessment and forecast of the effectiveness of various political measures to combat the shadow economy. It helps the government in choosing the most optimal strategies and programs. Simulations make it possible to assess the potential impact of government decisions on the level of the shadow economy and the country's economic stability. This is important to prevent negative consequences and minimize risks. With the use of mathematical models, it is possible to forecast the flow of taxes and other revenues from the shadow economy, which allows the government to more effectively plan the budget and allocate resources. The models make it possible to create a system for monitoring and evaluating the dynamics of the shadow economy, which allows you to respond to changes in a timely manner and introduce the necessary corrections into the policy. The use of mathematical modeling allows you to analyze the experience of other countries in the fight against the shadow economy and use the best practices to achieve positive results in Ukraine.



Source: author's own research.

Fig. 2. **Concept map on the relationship between the shadow economy and mathematical modeling**

From an economic point of view, mathematical modeling of the shadow economy is extremely important for Ukraine, as modeling of the shadow economy allows identifying potential risks for the economy of Ukraine, such as insufficient competitiveness, market instability or an unsatisfactory level of investment. Based on the results of mathematical modeling, it is possible to develop and implement effective strategies to combat the shadow economy, such as increasing fiscal discipline, creating a favorable business environment, and strengthening regulatory mechanisms. Modeling allows analyzing the effectiveness of measures aimed at stimulating the legal economy and reducing the shadow sector. This contributes to increasing investor confidence, increasing transparency and developing competitive business.

Understanding the dynamics. Dynamic models in mathematical modeling of the economy are used to study changes over time in economic phenomena and processes. Their main difference from static models is that they take into account the dynamics of changes in economic variables over time. Several dynamic models can be used to simulate the shadow economy, each of which offers a different insight into its dynamics and behavior.

The main components of dynamic models for the economic modelling are presented in Table 1.

Table 1

The main components of dynamic models

№	Component	Explanation
1	Time series	Dynamic models are based on time series of data describing changes in economic indicators over time. This data can be obtained from statistical sources or calculated on the basis of other data.
2	Differential Equations	Dynamic models often use differential equations to describe changes in variables over time. These equations can be linear or non-linear depending on the specific model.
3	Variable parameters	Some dynamic models take into account variable parameters that may change over time or depending on other factors.
4	Interpretation of dynamics	Dynamic models allow studying the dynamics of processes in the economy, such as changes in production, consumption, investment, etc. They help predict future trends and respond to economic changes.

Source: author's own research.

So, several types of dynamic models for modeling the shadow economy will be discussed below with simple possible ideas for examples.

1. Agent-based models (ABMs). ABMs model the behavior of individual economic agents, such as households, firms, and government agencies, in a virtual economy. These models can capture the complex interactions and feedback loops that contribute to the emergence and development of the shadow economy. Agent-Based Models (ABMs) simulate the behavior of individual agents and their interactions within a system. In the context of modeling the shadow economy using ABMs, it is needed to define the behavior and decision-making processes of agents regarding their participation in the formal and informal economies. For example, a small economy with agents making decisions about whether to participate in the formal economy (working in a registered business) or the shadow economy (engaging in informal or unregistered activities) [3]. Agents' decisions will be influenced by their expected utility, risk perception, and social interactions. The model has the following components:

Agent Characteristics	Each agent has income $Income_i$ and a risk aversion level $Risk_i$. Agents can choose between working in the formal sector or participating in the shadow economy.
Utility Function	The general utility function looks like this: $$U = U(Formal) + U(Informal) + OtherFactors$$ Utility functions can be based on economic theories such as utility maximization or behavioral economics concepts such as prospect theory. Agent *i*'s utility function could be: $$U_i = \alpha \cdot Income_i - \beta \cdot Risk_i + \gamma \cdot (1 - Compliance_i)$$ Where: - $Compliance_i$ is an indicator variable (0 or 1) representing whether the agent complies with formal sector regulations.

	- α, β, γ are parameters which represent the weights of income, risk aversion, and the cost of non-compliance, respectively.
Decision Rule	Agents make decisions based on rules or algorithms that consider their current state, information, and external factors: If $U(Informal) > U(Formal) \mid CostOfDetection$ agent participates in the shadow economy. For example, agents compare the expected realization of work in the formal sector with the shadow economy: If $U_t(F) > U_t(S) + CostOfDetection$ the agent chooses the formal sector *(F)*. Otherwise, the agent chooses the shadow economy *(S)*.
Inter-agent Interactions	Agents interact with each other based on a social network. Agents may share information about opportunities, risks, and enforcement levels. Social influence can affect agents' perceptions and decisions regarding informal sector participation.
Simulation Dynamics	Run the simulation for multiple time steps. Update agents' decisions based on changing economic conditions, enforcement levels, and interactions. Analyze the impact of policy changes, economic shocks, or behavioral factors on informal sector dynamics. Track the aggregate size of the shadow economy and formal economy over time.
Possible Simulation Steps	1) Initialize agents with random income levels, risk aversion, and compliance status. 2) Calculate each agent's utility for formal and shadow economy participation. 3) Agents make decisions based on their utility and the decision rule. 4) Agents interact, share information, and update their decisions accordingly. 5) Repeat steps 2–4 for multiple time steps to observe the dynamics of the shadow economy.

This example illustrates a basic ABM framework for modeling the shadow economy. Depending on the research objectives and complexity desired, additional factors such as enforcement dynamics, market competition, and government policies can be incorporated into the model.

2. Dynamic stochastic general equilibrium (DSGE) models. DSGE models are used to study the interaction between different sectors of the economy over time. They include decision-making processes, market dynamics and policy interventions to analyze how shocks or policy changes affect the size and persistence of the shadow economy. DSGE models typically represent economic agents (such as households and firms) as optimizing agents that make decisions to maximize their utility or profits, subject to various constraints and market conditions. These models include equilibrium conditions that ensure consistency between the decisions made by different agents and the overall functioning of markets. Dynamic Stochastic General Equilibrium models are used to study the behavior of the economy over time, incorporating stochastic shocks and optimizing agents' decisions [9]. Here are the main formulas and components used in DSGE models for modeling the shadow economy:

Households	Households in a DSGE model are modeled as consumers who allocate their income between consumption and saving based on their preferences and expectations about future income and economic conditions. They supply labor to firms in exchange for wages. The labor supply decision considers factors such as wage rates, non-labor income, and preferences for leisure versus work. Households may also make investment decisions, such as purchasing financial assets or real

estate, and managing their portfolios based on risk and return considerations. Household's Optimization Problem:

$$max_{\{C_t,N_t\}} U = \sum_{t=0}^{\infty} \beta^t \cdot u(C_t, N_t)$$

Subject to:

$$C_t + I_t = (1-\tau) \cdot Y_t + (1 + r_{t-1}) \cdot A_{t-1}$$
$$Y_t = F(K_t, N_t, A_{t-1})$$
$$\dot{A}_t = Y_t - C_t - I_t$$

Where:
- U represents the representative agent's utility.
- β is the discount factor representing time preference.
- $u(C_t, N_t)$ is the instantaneous utility function.
- I_t represents investment at time *t*.
- C_t represents consumption at time *t*.
- N_t represents labor supply at time *t*.
- τ is the tax rate.
- Y_t is output.
- $F(K_t, N_t, A_{t-1})$ is the production function.
- $\dot{A}_t$ is the asset level (the dot notation represents the change in assets over time).

Firms

Firms produce goods and services using inputs such as labor and capital. They make investment decisions based on factors like expected profitability, interest rates, and technological advancements. Firms set prices for their products, considering production costs, market demand, and competitive pressures. Price-setting behavior can influence inflation dynamics in the economy. Firms determine levels of employment and output based on demand conditions, production technology, and input costs. Their decisions contribute to aggregate supply and overall economic activity. Firm's Production Function:

$$Y_t = A_t \cdot K_t^{\alpha} \cdot N_t^{1-\alpha}$$

Where:
- Y_t is output.
- A_t represents total factor productivity.
- K_t represents capital.
- N_t represents labor supply.
- α is the capital share in production.

Government

The government in a DSGE model represents public sector activities such as taxation, government spending, and transfer payments. Fiscal policy decisions can impact aggregate demand, income distribution, and public debt levels. Central banks or monetary authorities are sometimes included as part of the government sector. They set monetary policy, including interest rates and money supply, to achieve macroeconomic objectives such as price stability and full employment. Governments may also implement regulations and policies affecting various sectors of the economy, such as labor market regulations, environmental standards, and industry-specific rules. Government Budget Constraint:

$$G_t + R_t \cdot B_{t-1} = T_t + \tau \cdot (Y_t - C_t) + B_t - B_{t-1}$$

Where:
- G_t is government spending.
- R_t is the interest rate.
- B_t represents government bonds.
- T_t is tax revenue.

Shadow Economy Dynamics

The shadow economy can be introduced as an unobserved sector in the model, affecting agents' decisions regarding labor supply, consumption, and savings. Its dynamics may depend on factors such as taxation, regulation, and enforcement. For example, let's introduce a simple shadow economy component influenced by taxation (ι) and regulatory environment (γ):

$$SE_t - \gamma(\iota \cdot Y_t)$$

Where:
- SE_t represents the size or share of the shadow economy at time *t*.
- γ captures the impact of taxation on the shadow economy.
- $\tau \cdot Y_t$ represents the tax revenue collected from the formal economy.

Stochastic Shocks

DSGE models often include stochastic shocks to capture random fluctuations in the economy. These shocks can affect variables such as productivity, preferences, or policy variables, influencing the behavior of agents and the overall economy.

$$A_t = \rho \cdot A_{t-1} + \epsilon_t$$
$$\epsilon_t \sim \mathrm{Normal}(0, \sigma^2)$$

Where:
- ϵ_t represents stochastic shock.
- A_t represents total factor productivity (TFP) affected by a stochastic shock.
- ρ represents the persistence of the shock.
- σ^2 is the variance of the shock.

Example Results (Hypothetical)

Suppose we simulate the model and find that an increase in taxation (τ) leads to a larger shadow economy due to tax avoidance behavior. Similarly, a positive stochastic shock (ϵ_t) in productivity boosts GDP and formal sector activities but may also impact the size of the shadow economy indirectly. This example demonstrates how a DSGE model can incorporate a shadow economy component to study the interplay between macroeconomic variables, policy measures, and unobserved economic activities over time [10].

These formulas outline the basic structure of a DSGE model and can be extended to incorporate specific features and dynamics related to the shadow economy. DSGE models are complex and typically solved numerically using simulation techniques to understand how various factors interact and impact economic outcomes over time.

3. Macroeconometric models. These models analyze macroeconomic factors affecting the shadow economy, such as GDP growth, unemployment, tax policy and the regulatory environment. They use econometric methods to estimate the relationships between these variables and the size of the shadow economy. In macroeconometric models, the shadow economy can be modeled as a function of various macroeconomic variables using regression equations [11]. Here are the main formulas and components used in macroeconometric models for modeling the shadow economy:

Basic Shadow Economy Equation

$$SE_t = f(GDP_t, Tax_t, Regulation_t, OtherFactors)$$

Where:
- SE_t represents the size or share of the shadow economy at time *t*.
- GDP_t is the real Gross Domestic Product at time *t*.
- Tax_t refers to the level of taxation or tax burden at time *t*.
- $Regulation_t$ represents the regulatory environment affecting formal and informal economic activities at time *t*.
- $OtherFactors$ may include variables such as unemployment rate, informal sector employment, income distribution, etc.

Regression Equation

For example, a typical approach in macroeconometric modeling is to estimate the shadow economy using regression analysis:

$$SE_t = \beta_0 + \beta_1 GDP_t + \beta_2 Tax_t + \beta_3 Regulation_t + \epsilon_t$$

Where:
- $\beta_0, \beta_1, \beta_2, \beta_3$ are coefficients representing the impact of GDP, taxation, and regulation on the shadow economy.
- ϵ_t is the error term capturing unexplained variation or other factors influencing the shadow economy not included in the model.

Dynamic Formulation

In dynamic macroeconometric models, lagged variables and time trends may be included to capture the persistence and dynamics of the shadow economy:

$$SE_t = \beta_0 + \beta_1 GDP_t + \beta_2 Tax_t + \beta_3 Regulation_t + \gamma SE_{t-1} + \delta t + \epsilon_t$$

Where:
- γ captures the lagged effect of the shadow economy.
- δt represents a time trend variable to capture long-term changes.

Error Correction Model (ECM)

In some cases, an Error Correction Model can be used to model the short-run dynamics and long-run equilibrium relationship between the shadow economy and its determinants:

$$\Delta SE_t = \alpha(SE_{t-1} - SE^*_{t-1}) + \beta \Delta GDP_t + \gamma \Delta Tax_t + \delta \Delta Regulation_t + \epsilon_t$$

Where:
- Δ represents first differences.
- SE^*_{t-1} represents the equilibrium level of the shadow economy.
- α captures the speed of adjustment towards equilibrium.

Estimation

Using historical data, estimate the coefficients ($\beta_0, \beta_1, \beta_2, \beta_3$) of the regression model through econometric techniques like ordinary least squares (OLS):

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n + \varepsilon$$

Where:
- Y is the dependent variable (response variable).
- $X_1 X_2, \ldots, X_n$ are the independent variables (predictor variables).

- $\beta_0, \beta_1, \ldots, \beta_n$ are the coefficients (parameters) to be estimated.
- ε is the error term representing the difference between the observed and predicted values.

Interpretation — Interpret the estimated coefficients to understand the relationship between GDP, taxation, regulation, and the shadow economy. Positive coefficients indicate a positive relationship (higher GDP, higher taxation, or more regulation leads to a larger shadow economy), while negative coefficients indicate the opposite.

Model Evaluation — Assess the goodness of fit of the model using statistical measures like R-squared, adjusted R-squared, and residual analysis. Test for the statistical significance of coefficients to ensure their reliability.

These formulas provide a foundation for modeling the shadow economy within macroeconometric frameworks. Depending on the specific context, additional variables, structural relationships, and time dynamics can be incorporated to improve the model's accuracy and predictive power.

4. Time series models. Time series analysis techniques such as autoregressive integrated moving average (ARIMA) models and vector autoregressive (VAR) models can be applied to historical data on economic indicators related to the shadow economy. These patterns help identify trends, seasonal patterns, and structural changes over time. Time series models are statistical models used to analyze and forecast time series data, where observations are collected at successive, evenly spaced intervals over time [12]. Here are the main formulas and components used in time series models for modeling the shadow economy:

Basic Time Series Model

$$SE_t = \mu + \varphi_1 SE_{t-1} + \epsilon_t$$

Where:
- SE_t represents the size or share of the shadow economy at time *t.*
- μ is the intercept term, representing the average level of the shadow economy.
- φ is the autoregressive coefficient, capturing the dependence of the current value on the previous value.
- ϵ_t is the error term at time *t*, assumed to be normally distributed with mean 0 and constant variance.

Autoregressive Integrated Moving Average (ARIMA) Model — Gather historical data on the size or share of the shadow economy (over multiple time periods. If the shadow economy data exhibits clear trends, seasonality, or autocorrelation, ARIMA models may be more appropriate. ARIMA models provide detailed parameter estimates that can aid in understanding the underlying dynamics of the shadow economy. For longer-term forecasting or capturing complex trends, ARIMA models may provide better accuracy than other time series models. ARIMA models are flexible and can capture complex patterns in the data, including trends, seasonality, and autocorrelation. They allow for differencing to handle non-stationarity, which is common in economic time series data. An ARIMA model combines autoregressive (AR), differencing (I), and moving average (MA) components:

$$\left(1 - \varphi_1 L - \varphi_2 L^2 - \cdots - \varphi_p L^p\right)(1 - L)^d\left(1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q\right)SE_t == \mu + \epsilon_t$$

Where:
- SE_t is the size of the shadow economy at time *t*.

	- p autoregressive order. - d is the differencing order (integrated component). - q is moving average order. - L is the lag operator. - $\varphi_1, \varphi_2, \dots, \varphi_p$ are autoregressive coefficients. - $\theta_1, \theta_2, \dots, \theta_q$ are moving average coefficients. - μ is the intercept term. - ϵ_t is the error term, assumed to follow a normal distribution.
Identifying Stationarity	Before applying an ARIMA model, ensure that the time series data of the shadow economy (SE_t) is stationary or can be made stationary through differencing. Stationarity implies that the statistical properties of the series (such as mean and variance) remain constant over time.
Estimation	Estimate the parameters of the ARIMA model ($\varphi_1, \varphi_2, \dots, \varphi_p, \theta_1, \theta_2, \dots, \theta_q, \mu$) using statistical methods such as maximum likelihood estimation (MLE) or least squares. Once the ARIMA model is estimated, the results can be used to forecast future values of the shadow economy. The next step is to generate forecasts and confidence intervals for the desired forecast horizon.
Model Diagnostics	The next step is to evaluate the goodness of fit of the ARIMA model using diagnostic tests and plots such as residuals analysis, ACF/PACF plots of residuals, and Ljung-Box test for autocorrelation in residuals. Than ensuring that the model assumptions (like normally distributed residuals and no autocorrelation in residuals) are met.

It is demonstrated how an ARIMA model can be applied to time series data, such as the size of the shadow economy, to capture and forecast its underlying patterns and behavior. Adjustments to the ARIMA model's parameters and orders can be made based on the data's characteristics and model diagnostics.

So, summing up all of the above, it is worth comparing the advantages and disadvantages of the types of models of dynamic modeling of the shadow economy. Table 2 provides a brief comparison of the advantages and disadvantages of each type of model. Each model has its strengths and weaknesses, and the choice of model depends on the specific research questions, data availability, and level of detail required for the analysis.

Table 2

Comparison of types of models for dynamic modeling of the shadow economy

№	Model Type	Advantages	Disadvantages
1	Agent-based models (ABMs)	1) Ability to simulate individual agent behavior and interactions. 2) Capture emergent phenomena and non-linear dynamics. 3) Flexibility in modeling heterogeneous agents with varied decision-making processes.	1) Complexity in modeling diverse agent behaviors. 2) Data-intensive, requiring detailed agent data. 3) Interpretability challenges due to complexity.

End of table 2

№	Model Type	Advantages	Disadvantages
2	Dynamic stochastic general equilibrium (DSGE) models	1) Incorporate microfoundations into macroeconomic models. 2) Suitable for analyzing economic policy impacts. 3) Capture interactions between different economic sectors.	1) Assumes rational expectations and perfect information, limiting realism. 2) Calibration challenges and sensitivity to parameter values. 3) Lack of financial sector detail in traditional DSGE models.
3	Macroeconometric models	1) Historical data-driven, providing empirical grounding. 2) Suitable for forecasting and policy analysis. 3) Can incorporate economic theories and structural relationships.	1) Limited ability to capture micro-level behaviors and interactions. 2) May oversimplify complex economic dynamics. 3) Sensitivity to data quality and assumptions.
4	Time series models	1) Focus on analyzing time-dependent data patterns. 2) Useful for short to medium-term forecasting. 3) Can identify trends, seasonality, and cyclical patterns.	1) Limited ability to capture complex interactions and feedback loops. 2) May not capture structural changes or long-term trends well. 3) Lack of explanatory power for underlying economic mechanisms.

Source: author's own research.

Therefore, mathematical modeling serves as a powerful tool in analyzing, understanding and solving problems related to the shadow economy, providing a quantitative framework for research, policy analysis and predictive analysis.

Conclusions. So, in this research work, four types of dynamic models for modeling the shadow economy were considered. Agent-based models (ABMs) offer a detailed perspective by simulating individual agent behaviors, albeit with complexity and data-intensive requirements. Dynamic stochastic general equilibrium (DSGE) models excel in incorporating microfoundations into macroeconomic analysis but may face challenges in calibration and assumption realism. Macroeconometric models provide empirical grounding and are suitable for forecasting and policy analysis, while time series models focus on time-dependent data patterns but may lack explanatory power for underlying economic mechanisms. Ultimately, the choice of modeling approach depends on the specific research objectives, available data, and desired level of detail. Integrating multiple modeling techniques or adopting hybrid models can mitigate the limitations of individual approaches and provide a more comprehensive understanding of dynamic shadow economy phenomena. Researchers and policymakers should carefully evaluate the strengths and weaknesses of each model type to effectively address the complexities of the shadow economy and inform evidence-based decision-making. Therefore, mathematical modeling has become a valuable tool in the study of the shadow economy, providing a quantitative framework for analyzing its size, dynamics, factors and consequences. As technologies and methodologies continue to develop, we can expect further progress and more effective applications of mathematical modeling in the future to understand and solve problems related to shadow economic activities. Understanding and addressing the shadow economy in Ukraine is crucial not only for maintaining economic stability, ensuring fair taxation, and promoting market integrity but also for improving governance, reducing informal employment, fostering social equity, and enhancing the country's international reputation and attractiveness for investors.

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